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ADAPTIVE CONTROL FOR A MECHANICAL SYSTEM WITH OSCILLATION DISTURBANCE

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A two-stage spring–lumped mass system was designed and built to investigate the suppression of vibration amplitude and the coupling tracking control problems. Disregarding the non-linear factors and unknown parameters, the system mathematical model could be formulated by using the state variable technique. A corresponding observable discrete time model was identified from a modified recursive least squares method based on the input and output data of this system. Then a robust multi-variable adaptive control strategy with pole assignment structure was proposed to control this mechanical system. Experiments were performed to evaluate the feasibility of this active vibration control strategy and the influence of the parameters estimator on the control performance. The experimental results showed that this approach can effectively diminish the amplitude of vibration and overcome the system coupling effect to obtain accurate position tracking.

1. INTRODUCTION

All mechanical systems are subjected to excitations that induce vibrations in the system. In order to diminish these kinds of vibrations, vibration control has become an interesting research topic for improving the system performance. Initially, vibration control was sought by using passive elements such as springs and dampers. The system vibration level was controlled by increasing the stiffness or adding damping. Rouch [1] called this kind of arrangement using springs and dampers the "dynamic absorber". However, the application of a dynamic absorber is limited to a certain range of frequencies. Hence, active vibration control has gained significant interest.

One of the earliest steps towards active vibration control was taken to control the relative motion between a cutting tool and a workpiece. Comstock [2, 3] actuated a cutting tool by using a control scheme which was a function of the relative displacement between the cutting tool and the workpiece surface. This helped to improve the stability of the cutting process. Klein and Nachtigal [4, 5] presented theoretical and experimental results for active control of a boring bar. Experiments were performed on a lathe equipped with a pivoted boring bar controlled by an electrohydraulic servo system. The cutting interface of the machine tool/workpiece was controlled by indirect actuation. Ellis and Mote [6] designed a feedback vibration controller to control the vibration of a circular wood cutting saw. It is clear that the effect of these three approaches is actively to change the stiffness and damping characteristics of the system. A new approach using an active dynamic absorber was proposed by Tewani *et al*. [7] actively to control a lumped mass system. The combination of passive elements, active elements and an absorber mass was used to apply

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a controlling force on the main system such that it reduced the amplitude of vibration of the system.

Currently, most of the control algorithms used in industry are for single-input–single-output (SISO) systems. However, most industrial systems are multi-input–multi-output (MIMO); for example, robots, machine tools, etc. Although many systems can be decoupled into SISO systems in order to simplify the design and control, systems with a complicated coupling behavior are difficult to decouple. Hence the analysis and control of MIMO systems have become important research topics in the industrial application of modern control algorithms.

Thompson and his co-workers [8] and Wilson *et al*. [9] used optimal control theory to control an active vehicle suspension system. Hac [10] employed the stochastic optimal control technique to investigate the suspension optimization problem of a two-degrees-of-freedom (2-DOF) vehicle model. Usually, industrial MIMO systems have non-linear time varying properties. In order to obtain better control performance, the gain parameters of the controller should be tunable against the time varying behavior of the system model. Adaptive control strategies have been widely used for this purpose. Sunwoo [11] used a model reference adaptive control to improve the performance of the suspension system of a two-mass quarter-car model. Sachs [12] and Karnopp and Margolis [13] proposed adaptive solutions for vehicle suspension systems, with parameters changing with respect to variation in the road conditions and the vehicle velocity. Hac [14] proposed an adaptive control scheme for a 2-DOF vehicle model with active suspension.

Pole assignment self-tuning control can adjust controller gains based on parameter variations of the identified system model in order to obtain the required dynamic response and stability. Elliott [15] proposed a direct arbitrary adaptive pole placement method which did not need to consider the behavior of zero cancellation. Mikles [16] combined the concepts of feedback, feedforward and precompensation to design a multi-variable self-tuning pole placement controller. Borrison [17] extended the minimum variance self-tuning control strategy to control multi-variable systems with the same numbers of inputs and outputs. Sinha [18] combined minimum variance with the decoupling concept to design a controller for multi-variable systems. Prager [19] proposed a self-tuning controller combined with the pole assignment method for multi-variable systems. That strategy can handle non-minimum phase systems with better robustness.

Most of the previous research on the control of multi-variable systems has been focused on theoretical study and computer simulation analysis. In this study, a simple MIMO system is designed to implement and evaluate the feasibility of a self-tuning controller. This active mechanical vibration system is shown in Figure 1. It is designed to simulate the simplified vehicle suspension system of a quarter-car model. The system analysis and control techniques discussed in this paper can be employed in industrial applications such as elevators and vehicle suspension systems. The DC servo motors serve as the active elements, which are installed on the external frame near the first and the secondary masses in order to provide active forces. Sinusoidal forcing input is applied to the system by a constant speed rotating wheel with an eccentric mass located on each stage.

2. THE MATHEMATICAL MODEL

A schematic diagram of the system under consideration is shown in Figure 1. The main mass with a gear rack is attached to the ground through a spring of stiffness K_1. It can slide on a guided pillar. A DC servo motor is installed on the external frame near the gear rack of each mass to control the motion of each stage through a rack-and-pinion gearing transmission system. A secondary mass is connected to the main system through a spring with stiffness K_2, which can slide also on the guided pillar. In addition, two wheels with eccentric masses driven by DC servo motors are installed on the main and the secondary masses to provide disturbance forces. Therefore, this system has four voltage control inputs of DC motors and four encoders to provide position outputs. In this study, the vibration system is considered as a two-input–two-output system with two internal excitation sources. Disregarding the non-linearities of the buckling deformation of the unguided spring and saturation of the control voltage, the dynamic equations of this MIMO system can be derived from Newton's second law as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{(k_1+k_2)}{\Omega_1} & -\dfrac{(B_1+B_2)}{\Omega_1} & \dfrac{k_2}{\Omega_1} & \dfrac{B_2}{\Omega_1} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_2}{\Omega_2} & \dfrac{B_2}{\Omega_2} & -\dfrac{k_2}{\Omega_2} & -\dfrac{B_2}{\Omega_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \dfrac{1}{\varepsilon\Omega_1} & 0 \\ 0 & 0 \\ 0 & \dfrac{1}{\varepsilon\Omega_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{u_{d1}}{\Omega_1} \\ 0 \\ \dfrac{u_{d2}}{\Omega_2} \end{bmatrix}. \qquad (1)$$

where $\Omega_1 = M_1 - \beta_1/\varepsilon$ and $\Omega_2 = M_2 - \beta_2/\varepsilon$, and x_1, x_3 and x_2, x_4 are the displacements and velocities of the main and the secondary masses, respectively. M_1 and M_2 are the masses

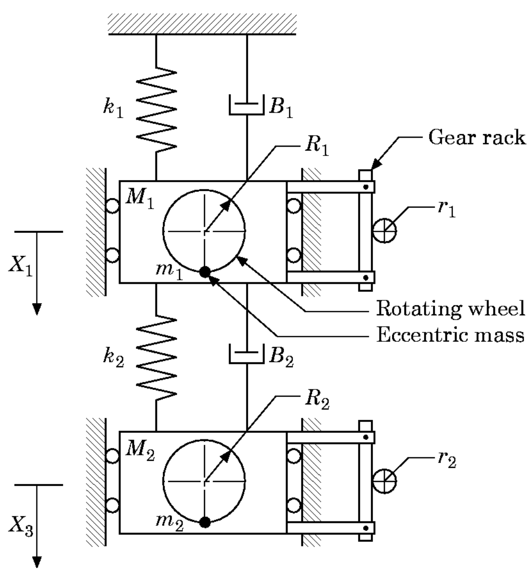

Figure 1. The schematic of the multi-variable vibration system.

of the main and the secondary stage, respectively. B_1 and B_2 are the damping coefficients of the main system and the secondary system sliding on the guide respectively. The inputs u_1 and u_2 are DC servo motor voltages. β_i, $i = 1, 2$, are constants which are functions of the gear radius, inertia and damping coefficient of the DC motor. The constant ε depends on the gear radius, the resistance, the damping coefficient and the torque constant of the DC motor. u_{d1} and u_{d2} are the disturbance forces applied to the main and the secondary systems respectively. The sinusoidal excitation forces are functions of the eccentric mass and the control voltage of the driving motor.

The above equation can be rewritten in matrix form as

$$\dot{X} = A_0 X + B_0 U + v. \tag{2}$$

Since some of the physical parameters in equation (1) are unknown and are difficult to estimate, this model cannot be used to design a controller or evaluate dynamic performance. In addition, the DC motors used in this system have position encoders only, without tachometer feedback. Hence, two position outputs only of the four states of this system are observed. For digital control implementation, an observable state space form with specified position feedback variables is chosen as a discrete time model to facilitate state estimation via an identification technique.

This mechanical vibration system is a coupled fourth order system with two position outputs and two control inputs. The interacting effect between both stages is due to the output coupling. Disregarding the non-linear behavior of the spring buckling deformation and control voltage saturation and the disturbance effect of sinusoidal excitation, one of the appropriate difference equations of this system was selected as

$$A(q^{-1})Y(k) = B(q^{-1})U(k) \tag{3}$$

where q^{-1} is the shifting operator and

$$A(q^{-1}) = \begin{bmatrix} 1 + a_{11}q^{-1} + a_{12}q^{-2} & a_{21}q^{-1} + a_{22}q^{-2} \\ a_{31}q^{-1} + a_{32}q^{-2} & 1 + a_{41}q^{-1} + a_{42}q^{-2} \end{bmatrix},$$

$$B(q^{-1}) = \begin{bmatrix} b_{11}q^{-1} + b_{12}q^{-2} & b_{21}q^{-1} + b_{22}q^{-2} \\ b_{31}q^{-1} + b_{32}q^{-2} & b_{41}q^{-1} + b_{42}q^{-2} \end{bmatrix}.$$

The above equation can be rewritten as the autoregressive and moving-average (ARMA) model form

$$\begin{aligned} Y(k) &= \theta(k-1)\Psi(k), \\ \Psi^{\mathrm{T}}(k) &= [-y_1(k-1) \quad -y_1(k-2) \quad -y_2(k-1) \quad -y_2(k-2) \quad u_1(k-1) \\ &\qquad u_1(k-2) \quad u_2(k-1) \quad u_2(k-2)] \end{aligned}$$

$$\theta(k-1) = \begin{bmatrix} \theta_1(k-1) \\ \theta_2(k-1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{21} & a_{22} & b_{11} & b_{12} & b_{21} & b_{22} \\ a_{31} & a_{32} & a_{41} & a_{42} & b_{31} & b_{32} & b_{41} & b_{42} \end{bmatrix}. \tag{4}$$

Least squares with a forgetting factor is a popular system identification algorithm which has good parameter convergence and is suitable for on-line parameter estimation. However, using this method it is possible to introduce output bursting behavior in the event of its being used in set point tracking control. Since the estimation process for a long period set point regulation does not have enough new input information, the adaptivity of an estimator without persistent excitation will deteriorate. That will cause an increase in the projection operator $P(t)$ and induce system uncertainty. Then the identified system parameters will vary violently once a new command input appears [20]. Therefore, the system output may experience bursting phenomena. The following methods are proposed to take care of this problem. One approach is to control the variation of the projection operator by limiting its value in a bounded range [21, 22]. The other approach is to employ a variable forgetting factor to follow the system changes [23]. Yet another approach is to combine both methods [24, 25]. The algorithm used in this study to identify the system time varying parameters $\theta_i(k)$, $i = 1, 2$, separately is [26]

$$\theta_i(k) = \theta_i(k-1) + a(k)K_i(k)e_i(k), \tag{5}$$

$$e_i(k) = y_i(k) - \hat{y}_i(k) = y_i(k) - \theta_i(k-1)\Psi(k), \tag{6}$$

$$K_i(k) = P_i(k-1)\Psi(k)(I + \Psi^{\mathrm{T}}(k)P_i(k-1)\Psi(k) + \bar{c}\Psi^{\mathrm{T}}(k)\Psi(k))^{-1}, \tag{7}$$

$$\begin{aligned}\bar{P}_i(k) = \bar{P}_i(k-1) - a(k)P_i(k-1)\Psi(k)[I + \Psi^{\mathrm{T}}(k)P_i(k-1)\Psi(k)\\ + \bar{c}\Psi(k)\Psi^{\mathrm{T}}(k)]^{-1}\Psi^{\mathrm{T}}(k)P_i(k-1),\end{aligned} \tag{8}$$

$$P_i(k) = C_1\frac{\bar{P}_i(k)}{\operatorname{tr}(\bar{P}_i(k))} + C_2 I, \tag{9}$$

$$a(k) = \begin{cases}\bar{a}, & \text{if } |y_i(k) - \theta_i(k-1)\Psi(k)| > 2\delta,\\ 0, & \text{otherwise.}\end{cases} \tag{10}$$

where $e_i(k)$ is the output prediction error of each shaft and $\Psi(k)_{8\times 1}$ is the regression vector. $P(k)_{8\times 8}$ is the projection operator and $\operatorname{tr}(P(k))$ is the trace of the projection operator. δ is the threshold value used to adjust the time varying parameter vector $\theta_i(k)_{1\times 8}$ when a significant error in the system output prediction has occurred. This algorithm forces the matrix $P(k)$ to stay within a certain range in order to prevent divergence and violent parameter variation due to system uncertainty. The purpose of using the variable $a(k)$ is to introduce a dead zone in the estimator. When the error of the output prediction is small, or the system does not have enough persistent excitation, it can reduce the sensitivity of the parameter estimation and eliminate the drifting phenomena of the system parameters. Hence, it can increase the robustness of the control system with respect to sudden variations in subsequent parameters.

The convergent speed of the system parameters is the most important factor influencing the performance of adaptive control. In order to reduce the initial error in the transient response, the initial values of the control system parameters are set to those obtained from an off-line system identification method which is part of the commercial software PC Matlab. In this study the inputs are a ± 5 V binary pseudo-random signal sequence. The outputs are the linear displacement of each mass. The parameter values of this vibration system, identified from an off-line least squares with the forgetting factor method, are listed in Table 1. The sampling frequency is 400 Hz. Since these system parameters have a time varying behavior and depend fully on the type and order of the selected difference ARMA model, the parameter values of each identification have different results. The parameters a_{ij} may have about a 30% drifting change and b_{ij} may have a drifting change between $-0{\cdot}02$ and $0{\cdot}04$ with respect to the corresponding parameters estimator and the command input change. The output responses of the first and the secondary masses of the real system and the identified discrete model are shown in Figure 2 for comparison. The solid line shows the output response of the real system and the dashed line depicts the output response of the identified model. The dynamic characteristics between them are matched

TABLE 1

The identified parameter values of the MIMO system

System parameters	Converged value	System parameters	Converged value
a_{11}	−1·9641	a_{31}	−0·0020
a_{12}	0·9687	a_{32}	0·0070
a_{21}	−0·0022	a_{41}	−1·9683
a_{22}	0·0020	a_{42}	0·9812
b_{11}	0·0134	b_{31}	0·0008
b_{12}	−0·0150	b_{32}	0·0021
b_{21}	0·0020	b_{41}	0·0150
b_{22}	0·0070	b_{42}	−0·0127

very well, except that deviation occurs at the peaks of the response curves due to the non-linear effect. Since the identified parameters cannot be ascertained as the true values of the system parameters, the interaction level of this system cannot be explained definitely on the basis of these data. An on-line parameter identification algorithm is required in the control loop to take care of the system dynamic characteristics. For the self-tuning

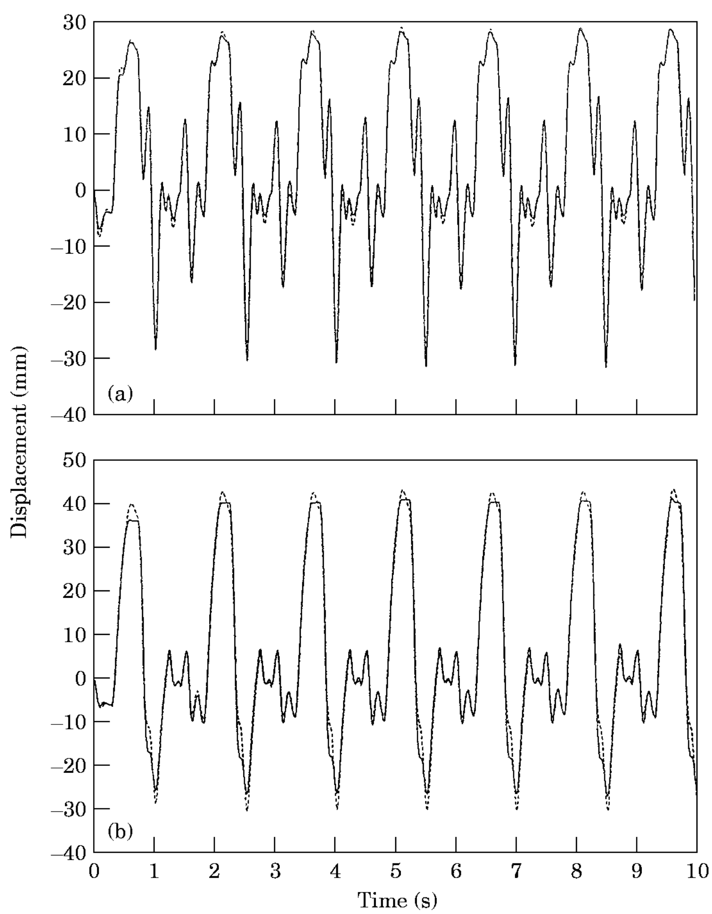

Figure 2. The output responses of the real system and the identified MIMO model. (a) The response of the main mass. (b) The response of the secondary mass.

adaptive control employed in this study, the parameters of this MIMO vibration system with fourth order dynamics are identified from equations (5)–(10). The system order of each subsystem of this two-mass coupling system is reasonably specified as 2, to synthesize a fourth order system model. From a system order point of view, a higher order model will not result in a significant improvement in the system responses. However, the computational cost will be increased very rapidly.

3. SELF-TUNING ADAPTIVE CONTROL

According to the system identification results, it was found that this mechanical vibration system has non-linear coupling and time varying behavior. Since the performance of a traditional controller depends on the accuracy of the system parameters, a self-tuning adaptive controller is employed to take care of the system time varying change. The identified parameters, a_{ij} and b_{ij}, have obvious variation corresponding to different parameter estimators and the system command input. In order to obtain a robust control system, a parameter estimator, described in equations (5)–(10), is introduced into the self-tuning loop of the controller to track the parameter variation, and the stability property of pole assignment [26] is employed in the controller design for the desired dynamic performance.

This fourth order vibration system consists of two mass–spring stages coupling through springs. The fourth order characteristic equation of the desired closed loop poles can be selected as the product of two second order polynomials:

$$z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 = (z^2 + a_{m1} z + a_{m2})(z^2 + a_{m3} z + a_{m4}). \tag{11}$$

The characteristic polynomial matrix used in the control input calculation can be simplified to

$$A_m(z^{-1}) = \begin{bmatrix} 1 + a_{m1} z^{-1} + a_{m2} z^{-2} & 0 \\ 0 & 1 + a_{m3} z^{-1} + a_{m4} z^{-2} \end{bmatrix}. \tag{12}$$

The design strategy of this adaptive controller employs the pole assignment concept with output feedback to obtain the desired system stability and the desired transient response under a reference input signal.

By taking the Z transformation of equation (3), one obtains

$$A(z^{-1}) Y(k) = B(z^{-1}) U(k). \tag{13}$$

The design rule of the adaptive control used in this study is as follows:

$$\begin{gathered} T(z^{-1}) Y_r(k) - S(z^{-1}) Y(k) = R(z^{-1}) U(k), \\ R(z^{-1}) = I z^{-d} + R_1 z^{-d-1} + \cdots + R_{nr} z^{-nr}, \\ S(z^{-1}) = S_0 + S_1 z^{-1} + \cdots + S_{ns} z^{-ns}, \\ T(z^{-1}) = T_0 + T_1 z^{-1} + \cdots + T_{nt} z^{-nt}, \end{gathered} \tag{14}$$

where $Y_r(k)$ and $U(k)$ are the reference position input and control input vectors respectively, R_i, $i = 1, \ldots, nr$, is an $m \times m$ coefficient matrix, and S_j, $j = 0, \ldots, ns$, and T_k, $k = 0, \ldots, nt$, are $m \times r$ coefficient matrices. d is the time delay of the system response. For this two-input–two-output vibration system, the dimensions of the R, S and T matrices are 2×2. In order to simplify the controller design procedure and reduce the computing time, the polynomial matrix $T(z^{-1})$ is set equal to the polynomial matrix $S(z^{-1})$. Then the

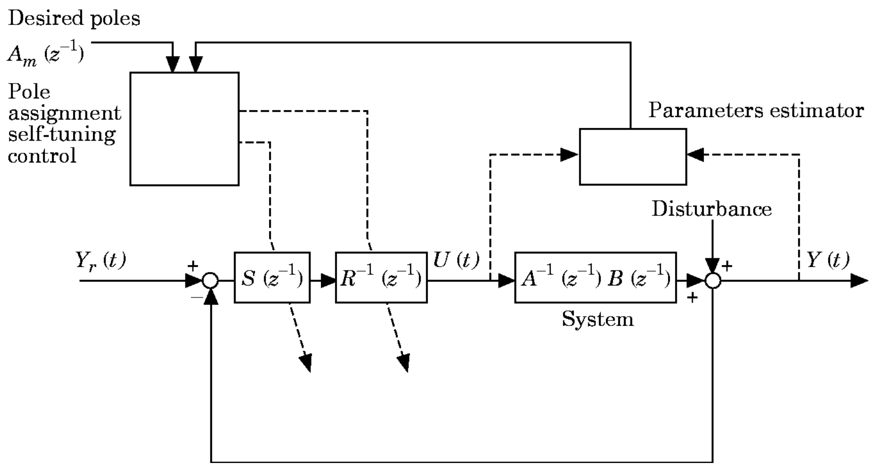

Figure 3. The block diagram of this pole assignment self-tuning control system.

pole placement self-tuning controller with unit feedback loop is shown in Figure 3. The orders of the polynomial matrices R and S are

$$nr = nb + d - 1 \qquad \text{and} \qquad ns = na - 1, \tag{15}$$

where na and nb are the orders of the polynomial in the system matrices A and B respectively. For this two-stage mechanical vibration system, R and S are given by

$$R(z^{-1}) = \begin{bmatrix} z^{-d} + r_{11}z^{-d-1} & r_{21}z^{-d} + r_{22}z^{-d-1} \\ r_{31}z^{-d} + r_{32}z^{-d-1} & z^{-d} + r_{41}z^{-d-1} \end{bmatrix}, \tag{16}$$

$$S(z^{-1}) = \begin{bmatrix} s_{10} + s_{11}z^{-1} & s_{20} + s_{21}z^{-1} \\ s_{30} + s_{31}z^{-1} & s_{40} + s_{41}z^{-1} \end{bmatrix}. \tag{17}$$

Since the matrix polynomial of the open loop transfer function is

$$G_0(z^{-1}) = A^{-1}(z^{-1})B(z^{-1})R^{-1}(z^{-1})S(z^{-1}), \tag{18}$$

then the characteristic polynomials of the open and the closed loop systems are

$$\alpha_0(z^{-1}) = \det\{A(z^{-1})\} \cdot \det\{R(z^{-1})\}, \tag{19}$$

$$\alpha_c(z^{-1}) = \det\{I + A^{-1}(z^{-1})B(z^{-1})R^{-1}(z^{-1})S(z^{-1})\}. \tag{20}$$

Assuming similar transformation relationship

$$R^{-1}(z^{-1})S(z^{-1}) = \tilde{S}(z^{-1})R^{-1}(z^{-1}), \tag{21}$$

where

$$ns = n\tilde{s} \qquad \text{and} \qquad \tilde{S}(z^{-1}) = \tilde{S}_0 + \tilde{S}_1 z^{-1} + \cdots + \tilde{S}_{ns}z^{-ns}.$$

The characteristic polynomial matrix equations of the closed loop system can be derived as follows:

$$\alpha_0(z^{-1})\alpha_c(z^{-1}) = \det\{A(z^{-1})R(z^{-1}) + B(z^{-1})\tilde{S}(z^{-1})\}. \tag{22}$$

If the characteristic matrix equation of the desired stable poles of the system is selected as

$$A_m(z^{-1}) = I + A_{m1}z^{-1} + \cdots + A_{mn}z^{-mn}, \tag{23}$$

then equality is obtained:

$$\det\{A_m(z^{-1})\} = \det\{A(z^{-1})R(z^{-1}) + B(z^{-1})\tilde{S}(z^{-1})\}. \tag{24}$$

Since equation (24) is a set of non-linear high order equations, their solutions are very difficult to solve. A solution can be obtained using the Diophantine equation:

$$A_m(z^{-1}) = A(z^{-1})R(z^{-1}) + B(z^{-1})\tilde{S}(z^{-1}). \tag{25}$$

For the existence of a solution to equation (25), the following condition must hold:

$$mn \leqslant \max\{(na + nr), (nb + d + n\tilde{s})\}. \tag{26}$$

By substituting equation (21) into equation (14), the stable inputs of the MIMO system can be calculated:

$$U(k) = \tilde{S}(z^{-1})R^{-1}(z^{-1})[Y_r(k) - Y(k)]. \tag{27}$$

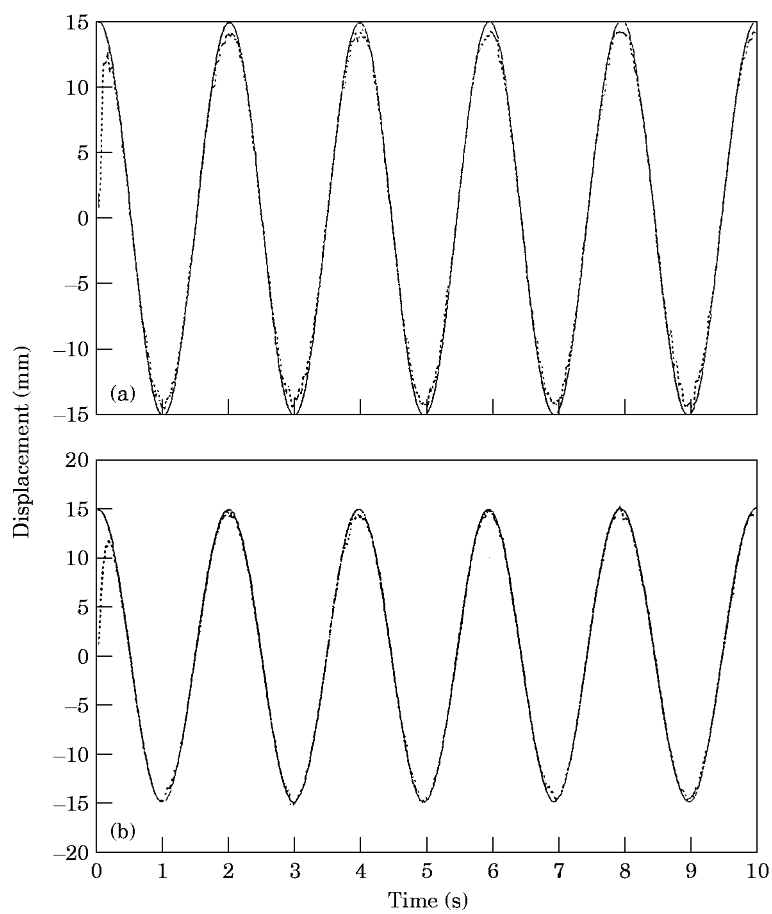

Figure 4. The output responses of sinusoidal waves by using a PID controller. (a) The main mass; (b) the secondary mass.

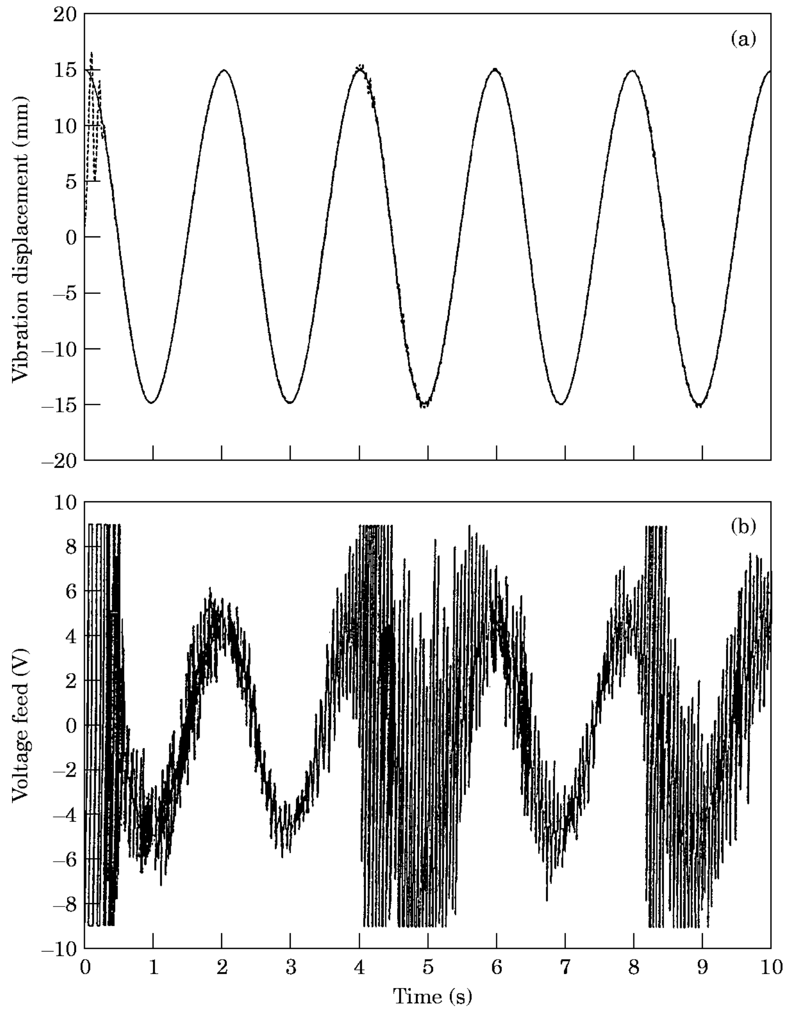

Figure 5. The output response (a) and the control input history (b) of the sinusoidal input tracking control of the main mass with least-squares identification.

The steps for this pole placement self-tuning control are as follows: (1) estimate the system parameters matrices $A(z^{-1})$ and $B(z^{-1})$ from equations (5)–(10); (2) solve equation (25) for the polynomial matrices equation $R(z^{-1})$ and $\tilde{S}(z^{-1})$; (3) calculate the control inputs from equation (27).

4. EXPERIMENTAL RESULTS AND ANALYSIS

The experimental layout of this active vibration control system consists of a microcomputer (IBM PC 80486-33) as the CPU of this digital controller, a DC servo motor actuated mass-spring structure, as in Figure 1, and two eight-bit interface cards with two sets of A/D and D/A converters to handle I/O communications between the PC and the motors. The resolution of the DC motor encoder is 400 pulses per revolution. The control law is written as a C language software program. The sampling frequency is 400 Hz.

The performance of this pole placement self-tuning controller is evaluated by implementing it in a multi-variable vibration system. Before the experiments are carried out, the initial values of $P(0)$, $\theta(0)$ and the desired closed loop poles must first be selected. The initial values of the system parameters are selected as one of the off-line identified results. The desired closed loop poles are designed in the S plane, and then they are transformed to the Z plane. The damping factor ζ is selected based upon the integral time average error (ITAE) rule, to obtain an optimal transient response when the system is subjected to a step input response. The undamped natural frequency ω_n is designed according to computer simulation to adjust the control gains for the desired steady state. The initial value $P(0)$ is chosen upon the basis of the evaluation of a performance index, which includes maximum overshoot, rise time and settling time for a step input response. A larger value of $P(0)$ will cause a more rapid convergence speed and a greater oscillation during the learning period. According to our study [20], the ideal values are $\zeta = 0{\cdot}7$, $\omega_n = 200$ rad/s and $P(0) = 0{\cdot}1I$. The time delay d is chosen as 2 on the basis of experimental estimation. In the following, results from two experiments are shown to evaluate the performance of the proposed controller.

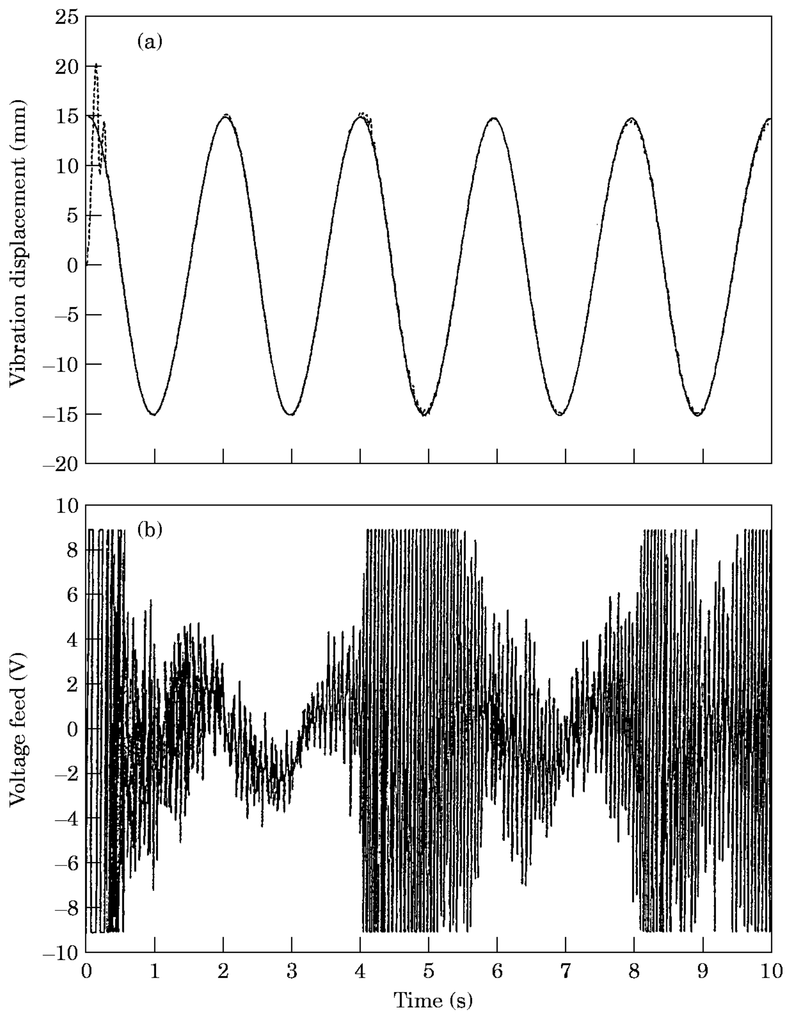

Figure 6. The output response (a) and the control input history (b) of the sinusoidal input tracking control of the main mass with least-squares identification.

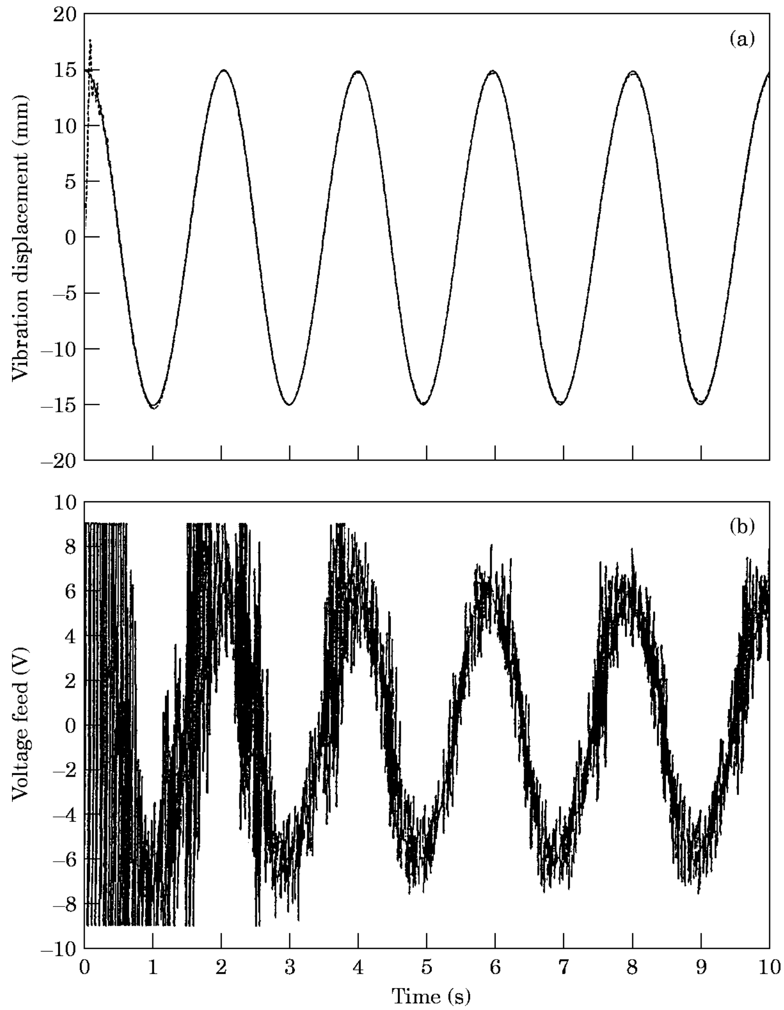

Figure 7. The output response (a) and the control input history (b) of the sinusoidal input tracking control of the main mass with modified least-squares identification.

4.1. TRAJECTORY TRACKING CONTROL WITHOUT DISTURBANCE

The specified motions of the main and the secondary masses are sinusoidal waves with 15 mm amplitudes. The output responses of the first and the second stages by using a PID controller are shown in Figures 4(a) and (b) respectively. The deviation from the reference trajectory at the peaks of the sinusoidal waves is due to the system non-linear and time varying effect. Hence, it is difficult for the PID controller implemented on this MIMO vibration system with time varying characteristics to obtain good control performance. In addition, the gains of the PID controller needs to be adjusted by trial-and-error for each control case. In order to eliminate the inconvenience and disadvantages of PID controller, a robust self-tuning adaptive control scheme is introduced in this study. The experimental results of self-tuning adaptive control with a fixed least squares forgetting factor of $\lambda = 0{\cdot}995$ for the sinusoidal inputs are shown in Figures 5 and 6 for the main and the secondary masses respectively. It can be observed that the deviation at the peaks of the sinusoidal waves is obviously improved. However, the calculated control input for the actuating motor is chattering between the upper and the lower bounds (± 9 V) and the system outputs are also chattering. This is the bursting phenomenon mentioned before. The reason is that the sinusoidal input is a persistent excitation signal with order 2 only, which is not a sufficiently persistent excitation signal for this fourth order system.

In order to improve the insufficient excitations of input signals, robust adaptive control strategies have been proposed by previous researchers [21, 22]. Here pole assignment self-tuning control with an identification scheme shown in equations (5)–(10) is employed for this multi-variable vibration system. The parameters used in equations (5)–(10) are $C_1 = 0{\cdot}001$, $C_2 = 0{\cdot}0$, $\bar{a} = 1{\cdot}0$, $\delta = 0{\cdot}0001$ and $P(0) = \overline{P}(0) = 0{\cdot}1I$. The output response and calculated input voltages are shown in Figures 7 and 8 for the main and the secondary masses respectively. Note that both the bursting phenomenon and the tracking performance are improved significantly. Since the actuating motor is not only used to actuate the system for the desired transient and steady state responses but is also utilized to overcome the inertia force in the gravity field and the tension or compression force of the coupling spring, high gains are required for ideal system performance. Hence, the calculated control input voltage has the feature of high frequency chattering. Its chattering

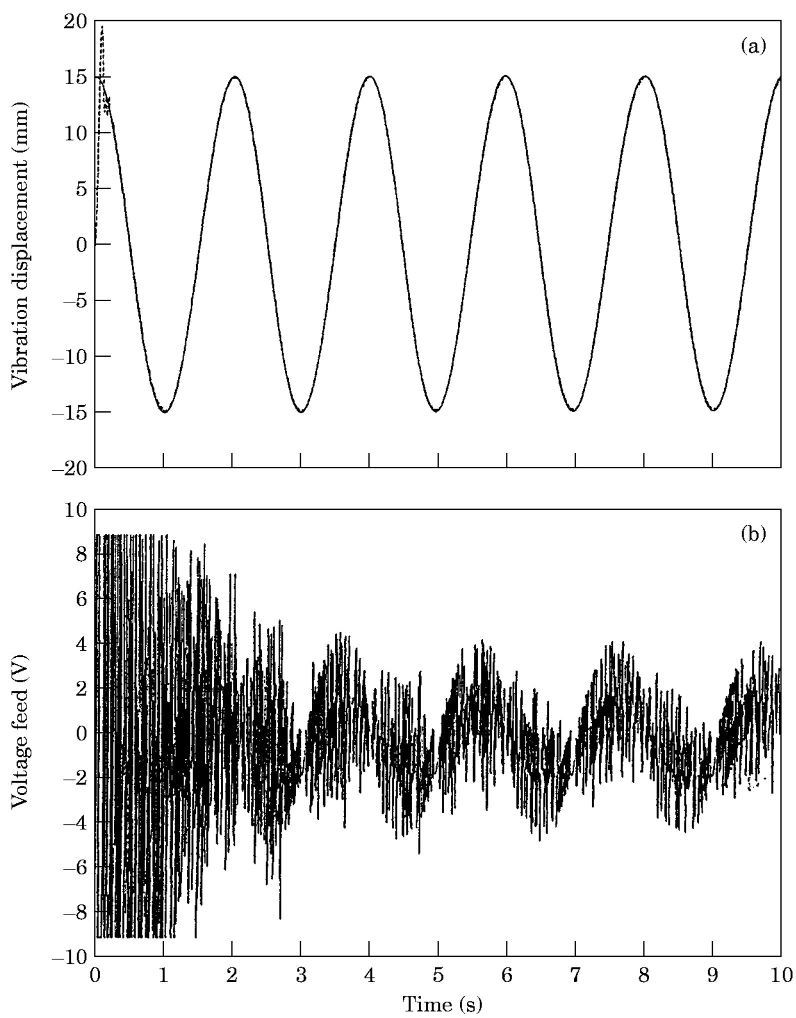

Figure 8. The output response (a) and the control input history (b) of the sinusoidal input tracking control of the secondary mass with modified least-squares identification.

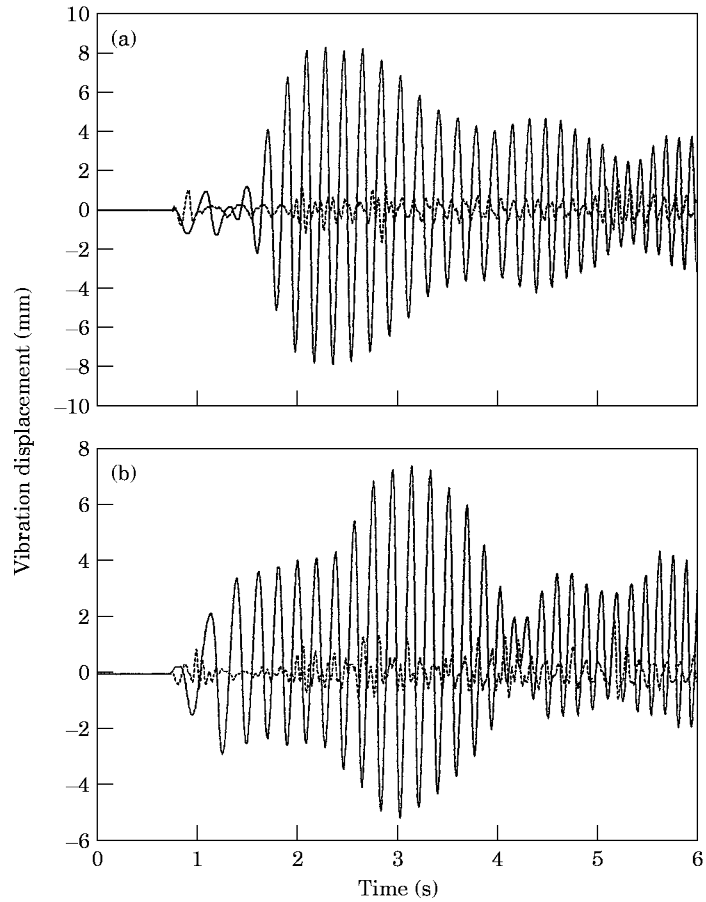

Figure 9. (a) The vibration suppression of the main mass. (b) The vibration suppression of the secondary mass under sinusoidal disturbance.

frequency depends on the gains and sampling frequency instead of the disturbance. If the order of the system controller is increased, the chattering characteristic does not improve significantly. However, the computational cost of this pole placement adaptive controller will be increased exponentially. It is not the correct choice for practical implementation.

4.2. VIBRATION SUPPRESSION UNDER DISTURBANCE

A sinusoidal disturbance is introduced into this vibration system by constant speed driving of the rotating wheel, with an eccentric mass located on each mass. The pole assignment self-tuning controller is employed to suppress the amplitude of vibration of each stage. Since this input disturbance is a persistent excitation, both least squares with a forgetting factor and the algorithm shown in equations (5)–(10) can be employed in the parameter identification loop. The amplitudes of vibration of the output response for the system under sinusoidal disturbance with constant frequency are shown in Figure 9. The solid line exhibits the system response without control action and the dashed line depicts the response of this system with the pole placement self-tuning control algorithm. The amplitude of vibration is reduced to about one sixth of its original value. The steady state behavior for the first 0·6 s is due to the starting delay of the exciting disturbance. The experimental suppression results for the system under random noise excitation are shown in Figure 10. It can be observed that the performance is also very good. If the vibration amplitude needs to be diminished furuther, stochastic adaptive control should be introduced by incorporating disturbance information into the controller design.

5. CONCLUSIONS

A pole placement self-tuning controller with a modified least squares identification algorithm is implemented on a two-stage multi-variable mechanical vibration system. Although the system exhibits non-linear time varying behavior, the experimental results show that the control performance of the trajectory tracking and the vibration suppression is satisfactory. The amplitude of vibration is reduced to about one sixth of the original value.

The desired poles are designed in the S-plane depending on the requirements of the system transient and steady state performance. The selection of an appropriate parameter estimator to obtain the best dynamic performance depends on the system input signals. When the system lacks sufficient excitation, a parameter estimator with less sensitivity is suitable. If the tracking input signal is a persistent excitation, the on-line estimator requires good adaptivity and sensitivity to diminish the residual error or model error. In order to

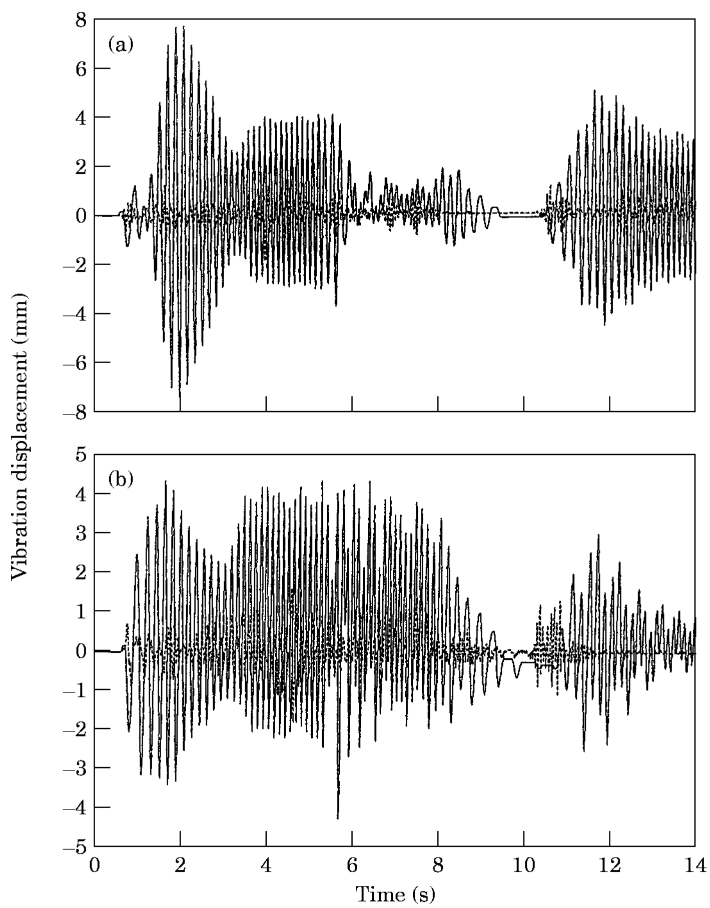

Figure 10. (a) The vibration suppression of the main mass. (b) The vibration suppression of the secondary mass under random noise excitation.

avoid the bursting phenomenon, a robust adaptive control with a modified least squares identification scheme is employed instead of the parameter identification scheme of least squares with a forgetting factor for the system without persistent excitation so as to eliminate the windup behavior of the estimated parameters.

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